Propagation of Rayleigh Waves in a Prestressed Layer over a Prestressed Half-space

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Abstract

This paper deals with the propagation of Rayleigh waves in a homogeneous, prestressed elastic layer of finite thickness over a homogeneous, prestressed elastic half-space. The dispersion equation has been derived for a layer over a half-space, when both media are considered as prestressed and the effect of initial stress shown in earlier investigators, is in general not applicable to the case of prestressed media, as well as the problem of Rayleigh waves has been solved by other method without introducing the displacement potentials.

The velocity equations for Rayleigh waves are obtained for two cases:

(I) in a homogeneous, prestressed elastic half-space without any superficial layer.

(II)in a homogeneous, elastic and finite layer over a homogeneous elastic half-space.

Keywords

Rayleigh Waves; Prestressed Media; Orthotropic Media; Anisotropic Media

Introduction

During earthquakes, the Rayleigh waves play more drastic role than other seismic waves in damages to human beings and buildings at the surface of the Earth. Therefore, it is of great importance to the seismologist to study the effect of initial stress on the propagation of Rayleigh waves. After the pioneering work of Rayleigh, many investigators have solved the Rayleigh problem for a half-space and one or more superficial layers situated over a half-space in homogeneous and non-homogeneous media. Most of the works related to the Rayleigh waves is well summarized in various books i.e. Achenbach, Ewing et al., Miklowitz, Ben-Menahem and Singh. However, all these studies have ignored the initial stress of the involved media. The model of the Earth is supposed to be composed of several layer of different thickness and inner layer of the Earth under immense stress owing to different physical causes i.e. presence of overlying layers, variation in temperature and gravitational field. Fortunately, Biot developed the incremental deformation theory for prestressed medium. Adapting the same theory of initial stress, Dey and Mukherjee solved the Rayleigh problem for a prestressed layer of compressible material and of finite thickness lying over an incompressible half-space under hydrostatic compressive stresses caused by gravity. The author solved this problem by introducing potential method, which is, in general, not applicable in prestressed media as shown by Sidhu and Singh.The same applies to the results obtained by Chattopadhyay et al. for the Rayleigh waves problems. In the present paper, our aim is to solve the problem firstly by direct method then potential method. The frequency equation of Rayleigh waves is derived from a homogeneous prestressed elastic layer over a homogeneous stressed elastic half-space. Here authors deduced two particular cases from this main frequency equation.

Firstly we obtain a frequency equation for Rayleigh waves in a prestressed half-space, which showed that this result is similar to the result obtained by Singh and Singh for equation of Rayleigh waves homogeneous. Secondly we deduced the expression for frequency equation of Rayleigh waves in a finite homogeneous elastic layer over a homogeneous elastic half-space from our main frequency equation by putting initial stress equal to zero. The derived result is similar to the result given by Ewing et al.

Basic Equations

Consider a homogeneous prestressed elastic layer of thickness H lying over a homogeneous, prestressed elastic half-space. The system is referred to a rectangular co-ordinate system with z-axis' directed normally downwards and the origin at the interface as shown in FIG. 1. Let the principal directions of initial stress be chosen to coincide with the directions of elastic symmetry and the co-ordinate axes. The state of initial stress is, therefore, defined by the principal components S_{11} , S_{22} *and* S_{33} of the initial stress. We restrict our analysis to plane strain parallel to the $x-z$ plane with displacements u - $w$in the x- z directions, respectively. The third principal stress $S_{22}=S_{33}$ does not enter explicitly into the equations of motion. Its influence is, however, included indirectly in the values of the incremental elastic coefficients which appear in the equations of motion. Let $S_{11} and S_{33}$ be the initial stresses along the x-z directions, respectively in the half-space and $S_{11}^{'} and\, S_{33}^{'}$ be the corresponding quantities in the layer. The equations of motion for incremental deformations in the half-space are given by Biot .

$$B_{11}\frac{\partial^2 u}{\partial x^2}+A_3\frac{\partial^2 w}{\partial x\partial z}+A_1\frac{\partial^2 u}{\partial z^2}=\rho\frac{\partial^2 u}{\partial t^2}$$

$$B_{22}\frac{\partial^2 w}{\partial z^2}+A_3\frac{\partial^2 u}{\partial x\partial z}+A_2\frac{\partial^2 w}{\partial x^2}=\rho\frac{\partial^2 w}{\partial t^2}\ , \tag{1}$$

Where $A_1=Q+\frac{P}{2}$, $A_2=Q-\frac{P}{2}$, $A_3=B_{12}+Q-\frac{P}{2}$, $P=S_{33}-S_{11}$.

(2)

For the layer, we denote the displacement by $(u^{'},v^{'},w^{'})$ the density by $\rho^{'}$ and the incremental elastic coefficients by $(B_{11}^{'},B_{22}^{'},B_{12}^{'},Q^{'})$. Then we can write the equations of motion for the layer similar to equation (1).

Boundary Conditions

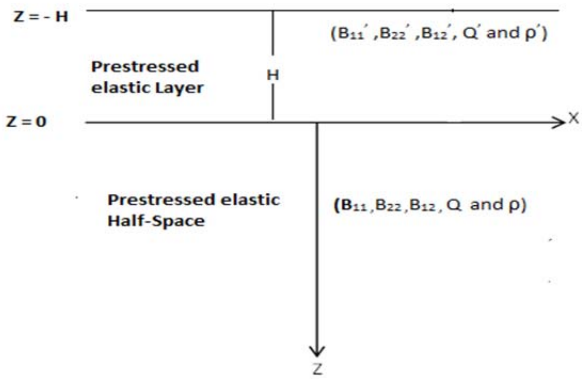

FIG. 1 GEOMETRY OF THE PROBLEM

For a layer with free surface at *z*= -*H*, the boundary conditions are

$$\Delta f_x^{'}=0,\ \Delta f_z^{'}=0 \text{ at } z=-H. \tag{3}$$

At the interface between layer and half-space at z = 0, the boundary conditions are

$$u=u^{'}, w=w^{'},\ \text{At } z=0 \tag{4}$$

$$\Delta f_x=\Delta f_x^{'}\ and\ \Delta f_z=\Delta f_z^{'}.$$

$\Delta f_x^{'}\ and\ \Delta f_z^{'}$ are the incremental boundary forces per unit initial area for the upper layer and the same for the half-space being $\Delta f_x\ and\ \Delta f_z$. The explicit expressions for $\Delta f_x^{'}, \Delta f_z^{'}, \Delta f_x$and $\Delta f_z\ are$

$$\Delta f_x^{'}=S_{31}^{'}+S_{33}^{'}\omega_y^{'}-S_{11}^{'}e_{zx}^{'}, \tag{5}$$

$$\Delta f_z^{'}=S_{33}^{'}+S_{33}^{'}e_{xx}^{'}\ , \tag{6}$$

$$\Delta f_z = S_{31} + S_{33}\omega_y - S_{11}e_{Zx}, \tag{7}$$

and

$\Delta f_z = S_{33} + S_{33}e_{xx}$. $S_{31}^{'}$, $S_{33}^{'}$, S_{31} , S_{33} are incremental stresses, $e_{zx}^{'}$, e_{xx} , e_{zx} , e_{xx} are incremental strainsand $\omega_y^{'}$, ω_y are the incremental rotation components parallel to the xz-plane. Explicit expressions of these quantities in terms of u and w are

$$S_{33}^{'} = \left(B_{12}^{'} - P^{'}\right)\frac{\partial u^{'}}{\partial x} + B_{22}^{'}\frac{\partial w^{'}}{\partial z}, \tag{8}$$

$$S_{31}' = Q'\left(\frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x}\right), \tag{9}$$

$$e_{xx}' = \frac{\partial u'}{\partial x},$$

$$e_{zx}' = \frac{1}{2}\left(\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z}\right), \tag{10}$$

$$\omega_y' = \frac{1}{2}\left(\frac{\partial u'}{\partial x} - \frac{\partial w'}{\partial z}\right), \tag{11}$$

with similar expression for S_{33} etc.

Propagation of Surface Waves

For Rayleigh waves of circular frequency ω, wave number k and phase velocity C_R propagating in the x-direction through the half-space, we may assume the solution of equation(1)

$$u = U_r e^{-kqz} e^{ik(C_R t - x)}, \tag{12}$$

$$w = W_r e^{-kqz} e^{ik(C_R t - x)}, \tag{13}$$

where U_r *and* W_r are the amplitude factors and q is assumed to be real and positive. Inserting the values of the displacements in equation (1), we get

$$(\rho C_R^2 - B_{11} + A_1 q^2)U_R + iqA_3 W_r = 0, \tag{14}$$

$$iqA_3 U_r + (\rho C_R^2 + B_{22}q^2 - A_2)W_r = 0. \tag{15}$$

The values of q may be obtained from

$$\begin{vmatrix} (\rho C_R^2 - B_{11} + A_1 q^2) & iqA_3 \\ iqA_3 & (\rho C_R^2 + B_{22}q^2 - A_2) \end{vmatrix} = 0 \tag{16}$$

On which expansion takes the form

$$q_1^2, q_2^2 = \frac{-[A_3^2 + \rho C_R^2(A_1 + B_{22}) - B_{11}B_{22} - A_1A_2] + [\{A_3^2 + \rho C_R^2(A_1 + B_{22}) - B_{11}B_{22} - A_1A_2\}^2 - 4\{A_1B_{22}(A_2B_{11} + \rho^2 C_R^4 - \rho C_R^2 A_2 - \rho C_R^2 B_{11}\}]^{1/2}}{2A_1B_{22}} \tag{17}$$

therefore the solution (13) can be written as

$$u = \left(U_{r_1}e^{-q_1kz} + U_{r_2}e^{-q_2kz}\right)e^{ik(C_R t - x)}, \tag{18}$$

$$w = \left(W_{r_1}e^{-q_1kz} + W_{r_2}e^{-q_2kz}\right)e^{ik(C_R t - x)}. \tag{19}$$

In the solution (13) of the equation of motion (1) for the half-space, we have considered only e^{-q_1kz} so that the radiation condition is satisfied. However, for the layer we must take both $e^{-kq^{'}z}$ and $e^{kq^{'}z}$ terms. For $e^{-kq^{'}z}$ the solution in layer is similar to equation (19). However, for $e^{kq^{'}z}$, we may assume

$$u^{'} = U_t^{'} e^{kq^{'}z} e^{ik(C_R t - x)}, \tag{20}$$

$$w^{'} = W_t^{'} e^{kq^{'}z} e^{ik(C_R t - x)}. \tag{21}$$

Inserting the values of the displacements in equation (1), we get

$$\left(\rho^{'} C_R^2 - B_{11}^{'} + A_1^{'} q^{'2}\right) U_t^{'} - iq^{'}A_3^{'}W_t^{'} = 0, \tag{22}$$

$$-iq^{'}A_3^{'}U_t^{'} + (\rho^{'}C_R^2 + q^{'2}B_{22}^{'} - A_2^{'})W_t^{'} = 0. \tag{23}$$

The values of $q^{'}$ may be obtained from

$$\begin{vmatrix} \rho' C_R^2 - B'_{11} + A'_1 q'^2 & -iq' A'_3 \\ -iq' A'_3 & \rho' C_R^2 + q'^2 B'_{22} - A'_2 \end{vmatrix} = 0 \qquad (24)$$

and

$$q^4 A_1 B_{22} + q^2 (A_3^2 + \rho C_R^2 A_1 + \rho C_R^2 B_{22} - B_{11} B_{22} - A_1 A_2) + (A_2 B_{11} + \rho^2 C_R^4 - \rho C_R^2 A_2 - \rho C_R^2 B_{11}) = 0 . \qquad (25)$$

This is a quadratic equation in q^2 *and* whose roots are $q_1^{'2}$, $q_2^{'2}$ similar to the values of q_1^2 *and* q_2^2 as in equation (17). Therefore the solution (19) can be written as

$$u' = \left(U'_{t_1} e^{kq'_1 z} + U'_{t_2} e^{kq'_1 z}\right) e^{ik(C_R t - x)}, \qquad (26)$$

$$w' = \left(W'_{t_1} e^{kq'_1 z} + W'_{t_2} e^{kq'_1 z}\right) e^{ik(C_R t - x)}. \qquad (27)$$

Surface for the upper layer

We may, therefore, assume the total displacement field to be the form in the layer

$$u' = \left(U'_{t_1} e^{kq'_1 z} + U'_{t_2} e^{kq'_1 z} + U'_{r_1} e^{-kq'_1 z} + U'_{r_2} e^{-kq'_2 z}\right) e^{ik(C_R t - x)}, \qquad (28)$$

$$w' = \left(W'_{t_1} e^{kq'_1 z} + W'_{t_2} e^{kq'_1 z} + W'_{r_1} e^{-kq'_1 z} + W'_{r_2} e^{-kq'_2 z}\right) e^{ik(C_R t - x)}, \qquad (29)$$

(U'_{t_1}, U'_{t_2}) , (W'_{t_1}, W'_{t_2}) , (U'_{r_1}, U'_{r_2}) , (W'_{r_1}, W'_{r_2}), C_R *and* k *have* conventional

meaning in the layer. $q_1^{'}$, $q_2^{'}$ are real and positive quantities.

$(U_{t_1}^{'}, U_{t_2}^{'})$ *and* $(W_{t_1}^{'}, W_{t_2}^{'})$ are not independent but connected by equation (23) for $q^{'} = q_1^{'}$ *and* $q_2^{'}$. Similarly $(U_{r_1}^{'}, U_{r_2}^{'})$ *and* $(W_{r_1}^{'}, W_{r_2}^{'})$ are also connected by equation (15) for $q^{'} = q_1^{'}$ and $q_2^{'}$. Taking second member of equations (23) and (15), we get

$$-iq_1^{'} A_3^{'} U_{t_1}^{'} + \left(\rho^{'} C_R^2 + q_1^{'2} B_{22}^{'} - A_2^{'}\right) W_{t_1}^{'} = 0 , \qquad (30)$$

$$-iq'_2 A'_3 U'_{t_2} + \left(\rho' C_R^2 + q_2'^2 B'_{22} - A'_2\right) W'_{t_2} = 0 , \qquad (31)$$

$$iq'_1 A'_3 U'_{r_1} + \left(\rho' C_R^2 + q_1'^2 B'_{22} - A'_2\right) W'_{r_1} = 0 , \qquad (32)$$

$$iq'_2 A'_3 U'_{r_2} + \left(\rho' C_R^2 + q_2'^2 B'_{22} - A'_2\right) W'_{r_2} = 0 , \qquad (33)$$

Equation (33) may be written as

$$U'_{r_1} = m'_1 W'_{r_1} \quad , U'_{r_2} = -m'_2 W'_r , \qquad (34)$$

$$U'_{t_1} = m'_1 W'_{t_1} \quad , U'_{t_2} = -m'_2 W'_{t_2} , \qquad (35)$$

where

$$m'_1 = iM'_1, m'_2 = iM'_2, \qquad (36)$$

and

$$M'_1 = \frac{\rho' C_R^2 + q_1'^2 B'_{22} - A'_2}{q'_1 A'_3} ,$$

$$M'_1 = \frac{\rho' C_R^2 + q_2'^2 B'_{22} - A'_2}{q'_2 A'_3} . \qquad (37)$$

Solution for the half-layer

The displacement field in the half-space may be written in the form equation (11).

$$u = \left(U_{r_1}^{'} e^{-kq_1 z} + U_{r_2}^{'} e^{-kq_2 z}\right) e^{ik(C_R t - x)}, \qquad (38)$$

$$w = \left(W_{r_1}^{'} e^{-kq_1 z} + W_{r_2}^{'} e^{-kq_2 z}\right) e^{ik(C_R t - x)}. \qquad (39)$$

(U_{r_1}, U_{r_2}) *and* (W_{r_1}, W_{r_2}) are not independent but connected by equation (15) for $q = q_1$ and q_2. Taking second member of equation (15), we get

$$iq_1 A_3 U_{r_1} + (\rho C_R^2 + q_1^2 B_{22} - A_2) W_{r_1} = 0\,, \tag{40}$$

$$iq_2 A_3 U_{r_2} + (\rho C_R^2 + q_2^2 B_{22} - A_2) W_{r_2} = 0\,. \tag{41}$$

Equation (41) may be written as

$$U_{r_1} = m_1 W_{r_1}\,, U_{r_2} = m_2 W_{r_2}, \tag{42}$$

where

$$m_1 = iM_1\,, m_2 = iM_1 \tag{43}$$

and

$$M_1 = \frac{\rho C_R^2 + q_1^2 B_{22} - A_2}{q_1 A_3},$$

$$M_2 = \frac{\rho C_R^2 + q_2^2 B_{22} - A_2}{q_2 A_3}\ . \tag{44}$$

$\left(U_{r_1}, U_{r_2}, W_{r_1}, W_{r_2}\right), C_R$ *and* k have conventional meaning in the half-space. q_1 , $q_2$are real and positive quantities defined in equation (17). The total displacement field given in equations (29) and (39) must satisfy the boundary conditions equations (3) and (4). Making use of equation (34) and (42), we obtain

$$ae^{-kq_1^{'}H} W_{t_1}^{'} + be^{-kq_2^{'}H} W_{t_2}^{'} + ae^{-kq_1^{'}H} W_{r_1}^{'} + be^{-kq_1^{'}H} W_{r_2}^{'} + 0W_{r_1}^{'} + 0W_{r_2}^{'} = 0\,, \tag{45}$$

$$fe^{-kq_1^{'}H} W_{t_1}^{'} + ge^{-kq_2^{'}H} W_{t_2}^{'} - fe^{-kq_1^{'}H} W_{r_1}^{'} - ge^{-kq_1^{'}H} W_{r_2}^{'} + 0W_{r_1}^{'} + 0W_{r_2}^{'} = 0, \tag{46}$$

$$aW_{t_1}' + bW_{t_2}' + aW_{r_1}' + bW_{r_2}' + lW_{r_1} + mW_{r_2} = 0\,, \tag{47}$$

$$fW_{t_1}' + gW_{t_2}' - fW_{r_1}' - gW_{r_2}' + nW_{r_1} + rW_{r_2} = 0\,, \tag{48}$$

$$-m_1' W_{t_1}' - m_2' W_{t_2}' + m_1' W_{r_1}' + m_2' W_{r_2}' - m_1 W_{r_1} - m_2 W_{r_2} = 0\,, \tag{49}$$

$$W_{t_1}' + W_{t_2}' + W_{r_1}' + W_{r_2}' - W_{r_1} - W_{r_2} = 0\,, \tag{50}$$

where

$$a = -\left[\left(Q' + \frac{P'}{2}\right) m_1' q_1' + \left(Q' - \frac{R'}{2}\right) i\right],$$

$$= -\left[\left(Q' + \frac{P'}{2}\right) m_2' q_2' + \left(Q' - \frac{R'}{2}\right) i\right],$$

$$f = (B_{12}' + S_{11}') m_1' i + B_{22}' q_1'\ \ ,$$

$$g = (B_{12}' + S_{11}') m_2' i + B_{22}' q_2'\ \ ,$$

$$l = \left[\left(Q + \frac{P}{2}\right) m_1 q_1 + \left(Q - \frac{R}{2}\right) i\right],$$

$$m = \left[\left(Q + \frac{P}{2}\right) m_2 q_2 + \left(Q - \frac{R}{2}\right) i\right],$$

$$f = (B_{12} + S_{11}) m_1 i + B_{22} q_1,$$

$$r = (B_{12} + S_{11}) m_1 i + B_{22} q_1 \tag{51}$$

Eliminating $W_{t_1}^{'}, W_{t_2}^{'}, W_{r_1}^{'}, W_{r2}^{'}, W_{r_1}$ *and* W_{r_2}, we obtain the following frequency equation for Rayleigh waves in prestressed media.

$$\text{Det A} = 0 \tag{52}$$

where A

$$Det\ A = \begin{bmatrix} ae^{-kq_1' H} & be^{-kq_2' H} & ae^{kq_1' H} & be^{kq_2' H} & 0 & 0 \\ fe^{-kq_1' H} & ge^{-kq_2' H} & -fe^{kq_1' H} & -ge^{kq_2' H} & 0 & 0 \\ a & b & a & b & l & m \\ f & g & -f & -g & n & r \\ & -m_1' & -m_2' m_1' m_2' & -m_1 & -m_1 & \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Particular Cases

If the initial state is unstressed, the elastic coefficients for the half-space will become

$$B_{11} = B_{22} = \lambda + 2\mu, \quad A_1 = A_2 = Q = \mu\,,$$

$$B_{12} = \lambda\,, \quad A_3 = \lambda + \mu\,, R = P = O\,. \tag{53}$$

From equations (44) and (17), we get

$$M_1 = q_1\,, M_2 = \frac{1}{q_2}\,, \tag{54}$$

$$q_1^2 = \left(1 - \frac{C_R^2}{\beta^2}\right)\,, q_2^2 = \left(1 - \frac{C_R^2}{\alpha^2}\right), \tag{55}$$

respectively, where

$$\alpha^2 = \frac{\lambda + 2\mu}{\rho}\ and\ \beta^2 = \frac{\mu}{\rho}\,. \tag{56}$$

$\lambda\ and\ \mu$ are Lame's constant for the half-space in unstressed media .Similarly for the layer, corresponding quantities becomes

$$B_{11}^{'} = B_{22}^{'} = \lambda^{'} + 2\mu^{'}\,, A_1^{'} = A_2^{'} = Q^{'} = \mu^{'}$$

$$B_{12}^{'} = \lambda^{'}\,, A_3^{'} = \lambda^{'} + \mu^{'}, R^{'} = P^{'} = 0\,, \tag{57}$$

and

$$M_1^{'} = q_1^{'}, \quad M_2^{'} = \frac{1}{q_2^{'}}\,, \tag{58}$$

where

$$q_1^{'2} = \left(1 - \frac{C_R^2}{\beta^{'2}}\right), \quad q_1^{'2} = \left(1 - \frac{C_R^2}{\beta^{'2}}\right). \tag{59}$$

Using equations (53) – (59) in equation (51) becomes

$$a = -i\left(2 - \frac{C_R^2}{\beta^{'2}}\right)\mu^{'}, b = -2i\mu^{'}\,,$$

$$f = 2\left(1 - \frac{C_R^2}{\beta^{'2}}\right)^{1/2}\mu^{'}\,, g = \frac{\left(2 - \frac{C_R^2}{\beta^{'2}}\right)\mu^{'}}{\left(1 - \frac{C_R^2}{\beta^{'2}}\right)^{1/2}}\,,$$

$$l = i\left(2 - \frac{C_R^2}{\beta^2}\right)\mu\,, m = 2i\mu\,,$$

$$n = 2\left(1 - \frac{C_R^2}{\beta^{'2}}\right)^{1/2}\mu\,,\ r = \frac{\left(2 - \frac{C_R^2}{\beta^2}\right)\mu}{\left(1 - \frac{C_R^2}{\beta^{'2}}\right)^{1/2}}\,. \tag{60}$$

Rayleigh Waves in Prestressed Half-space

In the absence of any prestressed layer, only two boundary conditions will be satisfied by the solution of the half-space. These are $\Delta f_x = \Delta f_z = 0$. Eliminating W_{r_1} and W_{r_2} , the frequency equation may be written as $\begin{vmatrix} l & m \\ n & r \end{vmatrix} = 0\,,$

Which after simplification of whichthe form can be obtained

$$\left[\left(Q + \frac{P}{2}\right)m_1 q_1 + i\left(Q - \frac{R}{2}\right)\right][i(B_{12} + S_{11})m_2 + B_{22}q_2] - [i(B_{12} + S_{11})m_1 + B_{22}q_1]\left[\left(Q + \frac{P}{2}\right)m_2 q_2 + i\left(Q - \frac{R}{2}\right)\right] = 0 \tag{61}$$

This result is equivalent to the corresponding result of Singh and Singh for the frequency equation of Rayleigh waves in the prestressed, homogeneous elastic half-space.

Rayleigh Wave in Unstressed Media

when both media are unstressed i.e. $P = R = P^{'} = R^{'} = 0$. Making use of equations(53)-(36), the equation (52) takes the form

$$\text{Det B} = 0 \tag{62}$$

where B is

which is the frequency equation of Rayleigh waves in a layer over a half-space in the unstressed media. These results coincide with the results given by Ewing et al..

Conclusions

For a prestressed layer lying over a prestressed half-space, the propagation of Rayleigh waves is considered. It is noted that the frequency equation of Rayleigh waves are effected due to the initial stresses present in the equation. The particular result is compared with the standard result

$$Det\ B$$

$$= \begin{bmatrix} \left(2-\frac{C_R^2}{\beta'^2}\right)e^{-kH\left(1-\frac{C_R}{\alpha'^2}\right)^{1/2}} & \left(2-\frac{C_R^2}{\alpha'^2}\right)e^{-kH\left(1-\frac{C_R}{\alpha'^2}\right)^{1/2}} & -\left(2-\frac{C_R^2}{\beta'^2}\right)e^{kH\left(1-\frac{C_R}{\alpha'^2}\right)^{1/2}} & -\left(2-\frac{C_R^2}{\beta'^2}\right)e^{kH\left(1-\frac{C_R}{\beta'^2}\right)^{1/2}} & 0 & 0 \\ \left(2-\frac{C_R^2}{\alpha'^2}\right)^{1/2} e^{-kH\left(1-\frac{C_R}{\alpha'^2}\right)^{1/2}} & \left(2-\frac{C_R^2}{\beta'^2}\right)^{1/2} e^{-kH\left(1-\frac{C_R}{\beta'^2}\right)^{1/2}} & \left(2-\frac{C_R^2}{\alpha'^2}\right)^{1/2} e^{kH\left(1-\frac{C_R}{\alpha'^2}\right)^{1/2}} & \left(2-\frac{C_R^2}{\beta'^2}\right)e^{kH\left(1-\frac{C_R}{\beta'^2}\right)^{1/2}} & 0 & 0 \\ 1 & (1-\frac{C_R^2}{\beta'^2})^{1/2} & -1 & -(1-\frac{C_R^2}{\beta'^2})^{1/2} & 1 & (1-\frac{C_R^2}{\beta'^2})^{1/2} \\ \left(1-\frac{C_R^2}{\alpha'^2}\right) & 1 & (1-\frac{C_R^2}{\alpha'^2})^{1/2} & 1 & -(1-\frac{C_R^2}{\alpha'^2})^{1/2} & -1 \\ 2(1-\frac{C_R^2}{\alpha'^2}) & (2-\frac{C_R^2}{\beta'^2}) & 2(1-\frac{C_R^2}{\alpha'^2})^{1/2} & (2-\frac{C_R^2}{\beta'^2}) & -2\frac{\mu}{\mu'}(2-\frac{C_R^2}{\alpha'^2})^{1/2} & -\frac{\mu}{\mu'}(2-\frac{C_R^2}{\beta'^2}) \\ (2-\frac{C_R^2}{\beta'^2}) & (2-\frac{C_R^2}{\beta'^2})^{1/2} & -(2-\frac{C_R^2}{\beta'^2}) & -(2-\frac{C_R^2}{\beta'^2})^{1/2} & \frac{\mu}{\mu'}(2-\frac{C_R^2}{\beta'^2}) & 2\frac{\mu}{\mu'}(2-\frac{C_R^2}{\beta'^2})^{1/2} \end{bmatrix}$$

obtained by Singh and Singh for frequency equation in prestress half-space. The frequency equation of Rayleigh waves for homogeneous elastic layer over a homogeneous elastic half-space is obtained and compared with the result without stresses given by Ewing et al..

ACKNOWLEDGMENTS

The author is deeply grateful to Dr. SarvaJit Singh for helpful comments and discussion. Author is also thankful to U.G.C for providing minor research project No. F.No.8.3(121)/2011(Mrp/NRCB)

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